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**Exercise 1.**

(3 points)

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Prove the proposition on “The  $k$ -Subsets Relaxation”, slide 5). Tip: prove by induction over  $i$ ,  $0 \leq i \leq n$ , that  $result(s, \langle a_1, \dots, a_i \rangle)$  is defined, and equal to  $(s \cup \bigcup_{j=1}^i add(a_j)) \setminus \bigcup_{j=1}^i del(a_j)$ .

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**Exercise 2.**

(2=1+1 points)

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Consider parallel regression in the parallel Blocksworld (see “The  $k$ -Subsets Relaxation”, slide 13 for parallel regression, and slides 10 and 14 for the parallel Blocksworld operators and goal). Write up the fact sets  $Regress(G, \{movefromtable(B, D), movefromtable(C, A)\})$  and  $Regress(G, \{movefromtable(B, D), move(C, E, A)\})$ .

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**Exercise 3.**(5=2+3 points)

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Prove the following propositions:

- **Proposition 1** Let  $(P, A, I, G)$  be a STRIPS task, let  $s \subseteq P$ , and let  $\langle a_1, \dots, a_n \rangle$  be a sequence of actions so that  $s \subseteq \text{result}(I, \langle a_1, \dots, a_n \rangle)$ . Then, for  $r(s)$  as defined on “The  $k$ -Subsets Relaxation” slide 18, we have that  $r(s) \leq n$ .
- **Proposition 2** Let  $(P, A, I, G)$  be a STRIPS task, let  $s \subseteq P$ , and let  $\langle a_1, \dots, a_n \rangle$  be a sequence of actions so that  $s \subseteq \text{result}(I, \langle a_1, \dots, a_n \rangle)$ . Then, for  $r^k(s)$  as defined on “The  $k$ -Subsets Relaxation” slide 19, we have that  $r^k(s) \leq n$ .

In both proofs, you may use without proof the property that, for any  $s, s'$  where  $s \subseteq s'$ ,  $r(s) \leq r(s')$ . Tip: For the first proof, denote  $s_i := \text{result}(I, \langle a_1, \dots, a_i \rangle)$ ; you can then prove by induction over  $i$ ,  $0 \leq i \leq n$ , that  $r(s_i) \leq i$ . For the second proof, you can use a similar induction.

Note: For the first proof, use **only** the characterization of  $r(s)$  on “The  $k$ -Subsets Relaxation” slide 18; do **not** use its definition as solution distance in sequential regression, as per slide 17!

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**Exercise 4.**(5 extra points)

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Prove the following proposition:

**Proposition 3** Let  $(P, A, I, G)$  be a STRIPS task, let  $s \subseteq P$ , and let  $\langle a_1, \dots, a_n \rangle$  be a shortest sequence of actions so that  $s \subseteq \text{result}(I, \langle a_1, \dots, a_n \rangle)$ . Then, for  $r(s)$  as defined on “The  $k$ -Subsets Relaxation” slide 18, we have that  $r(s) \geq n$ .

This is a **voluntary** exercise, meaning that you can earn those 5 points, but the 5 points are **not** counted into the sum of achievable points. Tip: there is a relatively easy proof by contradiction.

Note: Use **only** the characterization of  $r(s)$  on “The  $k$ -Subsets Relaxation” slide 18; do **not** use its definition as solution distance in sequential regression, as per slide 17!