UNIVERSITY OF INNSBRUCK
MASTERSTUDIUM INFORMATIK VERTIEFUNGSMODUL (WM 9)
Automatic Planning
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Exercises Nr. 7. **To be returned on Tuesday, Dec 4**

Exercise 1. (3 points)

Prove the proposition ("The Monotonicity Relaxation", slide 36) showing that, for any k, in the k-planning graph ("The k-Subsets Relaxation", slide 41) for a relaxed STRIPS task (P, A^+, I, G) (all delete lists are empty), all sets EF_t and AF_t are empty. Tip: prove this by induction over t, base case $t = 0$. The proof fits easily into half a page. Don't write more.

Exercise 2. (4=2+2 points)

Consider the logarithmic STRIPS encoding of incrementing a variable x ("The Monotonicity Relaxation", slide 70). Precisely, the facts are $\{Tbit_0$, $Fbit_0$, $Tbit_1$, $Fbit_1$, $Tbit_2$, $Fbit_2$, $Tbit_3$, $Fbit_3\}$, wit the obvious meaning. The initial state is $\{Fbit_0$, $Fbit_1$, $Fbit_2$, $Fbit_3\}$, the goal is $\{Tbit_0$, $Tbit_1$, $Tbit_2$, $Tbit_3\}$. There are 15 actions $inc_0, \dots inc_{14}$; inc_i requires $x = i$ in the precondition, and sets $x := i + 1$, by the respective conditions and updates to the bits. For example, $inc_7 = (\{Tbit_0, Tbit_1, Tbit_2, Fbit_3\}$, $\{Fbit_0, Fbit_1, Fbit_2, Tbit_3\}$, $\{Tbit_0, Tbit_1, Tbit_2, Fbit_3\})$.

1. Build a relaxed planning graph for this task, i.e., write up the sets F_t and A_t until the algorithm ("The Monotonicity Relaxation", slide 36) terminates. Leave out the NOOP actions. Use F_t in the write-up of F_{t+1}, and A_t in the write-up of A_{t+1}.

2. Extract a relaxed plan ("The Monotonicity Relaxation", slide 39). Write up the contents of the sets $G_0, \dots, G_{maxlevel}$, before the 2nd **for**-loop ($t := maxlevel, \dots, 1$) starts, and at the end of every iteration of that loop; say what actions are selected in each iteration. What is the value of h^{FF}?

Exercise 3. (3 points)

Consider the following decision problem. Of n numeric variables $v_1, \dots, v_n$, each has $dom(v_i) = \{0, 1\}$, i.e., it can be set to either 0 or 1. A numeric expression $exp(v_1, \dots, v_n)$ is formed by $v_1, \dots, v_n$ and the standard operators $+, -, *, /$. The question to be answered is: *Given an arbitrary numeric expression, does there exist an assignment* $\{v_1 \mapsto c_1 \in AS(v_1), \dots, v_n \mapsto c_n \in AS(v_n)\}$ *to the variables so that the value of* $exp(c_1, \dots, c_n)$ *is greater than* 0*?*

Prove that this decision problem is **NP**-complete. Tip: use a polynomial reduction of SAT.

Note: this corresponds to a problem that must be answered as a sub-problem of relaxed numeric STRIPS with arbitrary expressions. Namely, $dom(v_i)$ corresponds to $AS(v_i)$, and answering the question is part of determining whether a relaxed action precondition is satisfied in a (relaxed) state. (See “The Monotonicity Relaxation”, slides 76 and 77.)