

## Automatic Planning

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- ▶ Success of heuristic search depends on quality of heuristic function
- ▶ Measure quality in terms of topological properties of the “search space surface” (local minima etc.)
- ▶ Here: FF and related planners extremely efficient in many planning benchmarks
- ▶ *Can we say something about the quality of these planners’ heuristic functions, in these benchmarks?*
- ▶ We focus exclusively on forward search in the state space



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Automatic Planning

Automatic Planning  
8. Local Search Topology

- ▶ **A Reminder:  $h^+$  and  $h^{FF}$**
- ▶ A Preview
- ▶ Local Minima & Co.
- ▶ 20 Planning Benchmark Domains
- ▶ Measuring Local Search Topology
- ▶ Proving Local Search Topology
- ▶ Sampling Local Search Topology



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 $h^+$  and  $h^{FF}$ 

## Definition

Let  $(P, A, I, G)$  be a STRIPS task,  $s$  a state in the state space.  $h^+(s)$  is the length of an optimal plan for  $(P, A^+, s, G)$ , or  $\infty$  if there is no such plan.

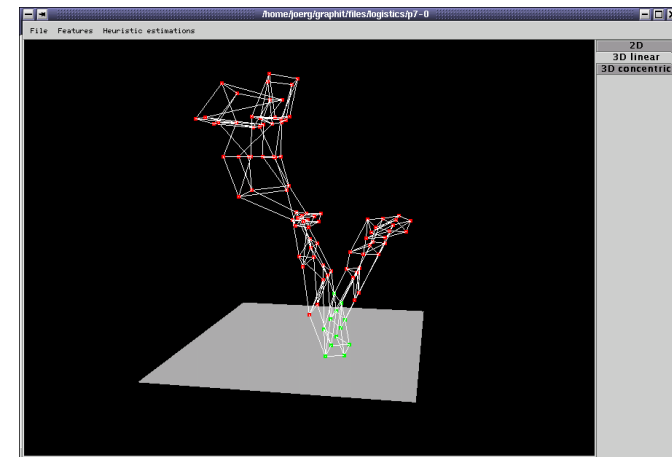
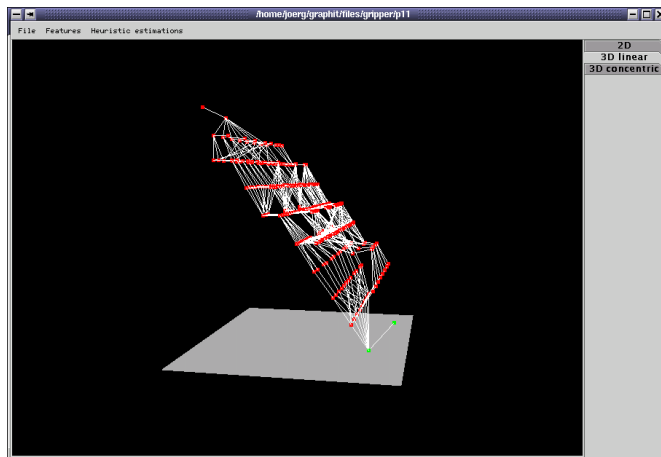
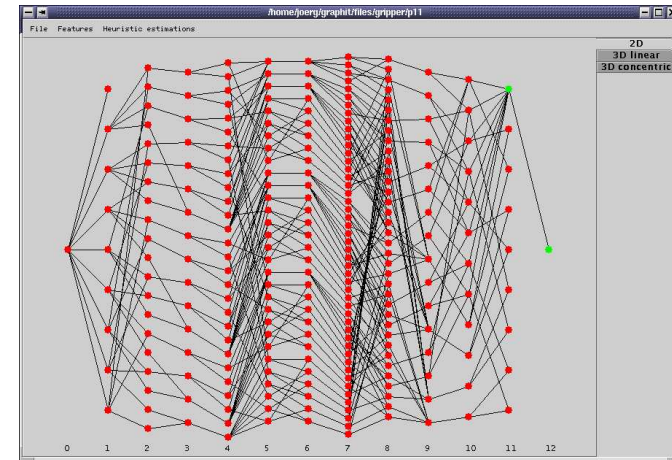
- ▶ Computing  $h^+$  is NP-hard
- ▶  $h^{FF}$  approximates  $h^+$  by a two-step (forward, backward) relaxed Graphplan process, computing some (not necessarily optimal) relaxed plan
- ▶ Related things are done in today’s fastest planners
- ▶ No provable connection between  $h^+$  and  $h^{FF}$  except for  $\infty$  (see later)



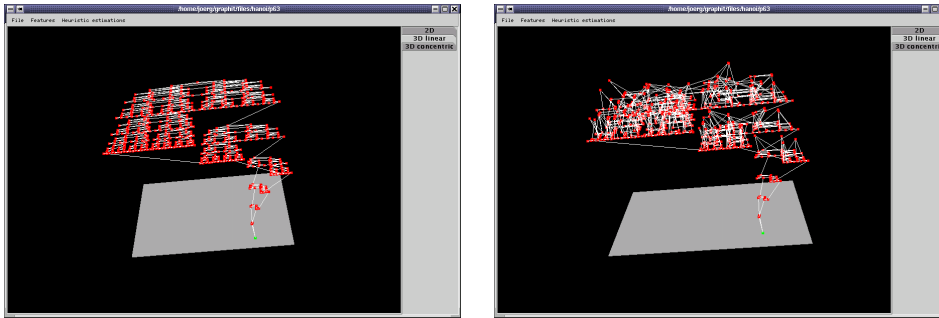
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Instance with 6 discs:



Can you guess what the explanation is?

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“Local Search Topology” [Frank et al,JAIR-1997] means topological phenomena of the state space graph annotated with heuristic values.

Here we’ll investigate:

- ▶ **Dead ends**
- ▶ **Local minima**
- ▶ **Benches**

We consider *solvable* tasks only. Why???

We denote with  $S$  the set of reachable states:

$$S := \{s \mid s = I \vee \exists s' \in S, a \in A : \text{result}(s', \langle a \rangle) = s\}$$

and with  $E$  the set of edges (action applications) between them:

$$E := \{(s', s) \mid \exists a \in A : \text{result}(s', \langle a \rangle) = s\}$$

*From what we know about  $I$  and  $G$ , what can we conclude about the graph  $(S, E)$ ?*



We annotate states  $s \in S$  with a heuristic value  $h(s)$ :

### Definition

Given a STRIPS task  $(P, A, I, G)$  and its state space graph  $(S, E)$ , a **heuristic function** is a function  $h : S \mapsto \mathbf{N}_0 \cup \{\infty\}$  so that  $h(s) = 0$  iff  $G \subseteq s$ .

That is, we assume that  $h$  is exact for goal states.

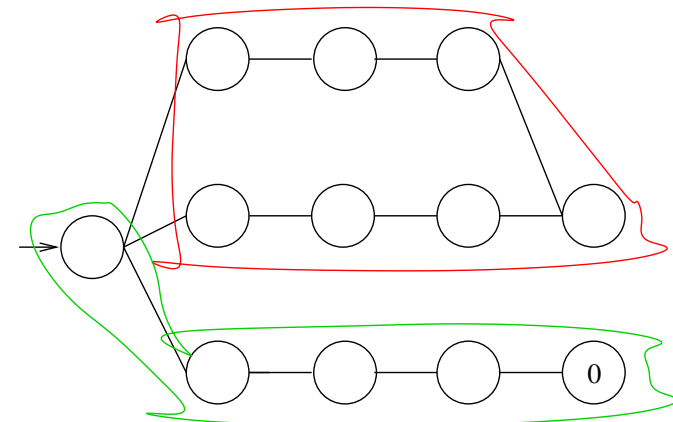
### Proposition

Given a STRIPS task  $(P, A, I, G)$  and its state space graph  $(S, E)$ , for all  $s \in S$  we have  $h^+(s) = 0$  iff  $h^{FF}(s) = 0$  iff  $G \subseteq s$ .

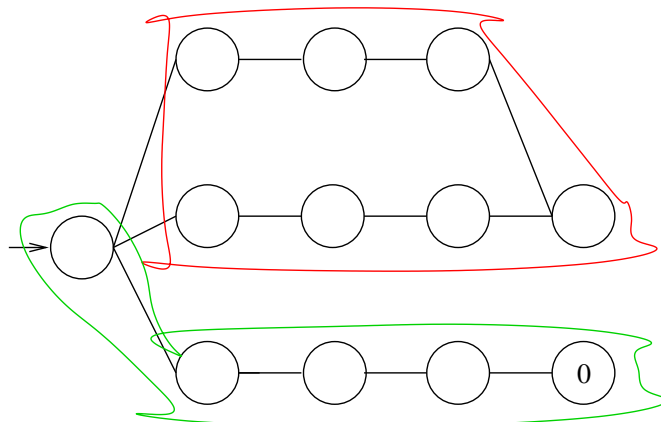
### Proof.

Do you see why? □

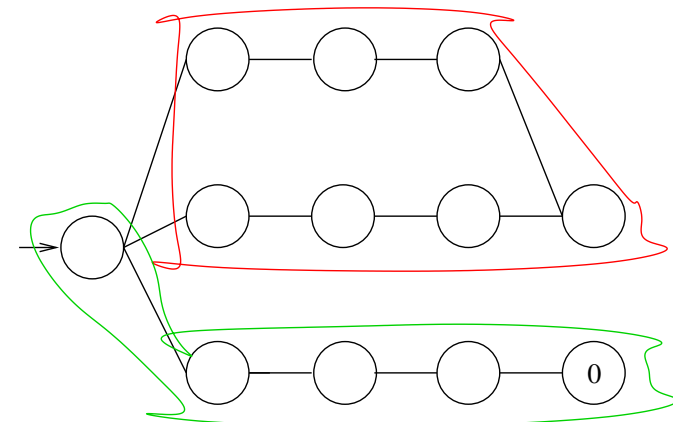
Dead ends are states from which no solution can be reached:



Local minima are regions where all neighbours look worse:



Benches are flat regions with better neighbours:



Definition

Let  $s \in S$  be a state.  $s$  is a **dead end** if  $sd(s) = \infty$ .

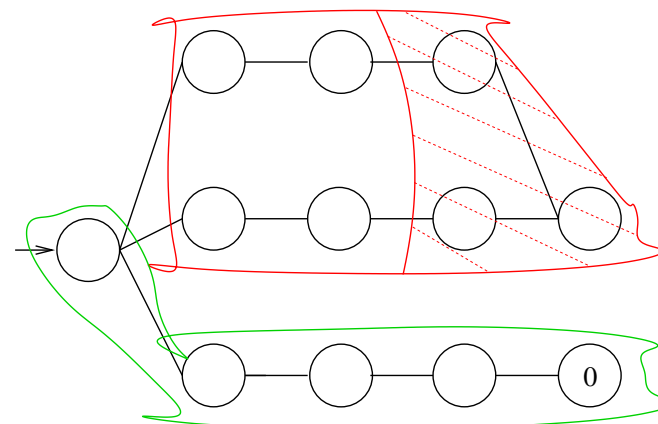
- ▶ Dead ends cannot arise if ???

Definition

Let  $s \in S$  be a dead end state.  $s$  is **recognized** if  $h(s) = \infty$ .

- ▶  $h$  returns  $\infty$  to indicate that  $s$  is a dead end
- ▶  $h$  is called **completeness-preserving** if  $h(s) = \infty \Rightarrow sd(s) = \infty$

Recognized and unrecognized dead ends:



Proposition

Let  $s \in S$  be a state. If  $h^+(s) = \infty$ , then  $s$  is a dead end.

Proof.

Do you see why? □

The same is true for any  $h$  based on solving a relaxation.

Proposition

Let  $s \in S$  be a state.  $h^+(s) = \infty$  iff  $h^{FF}(s) = \infty$ .

Proof.

Do you see why? □

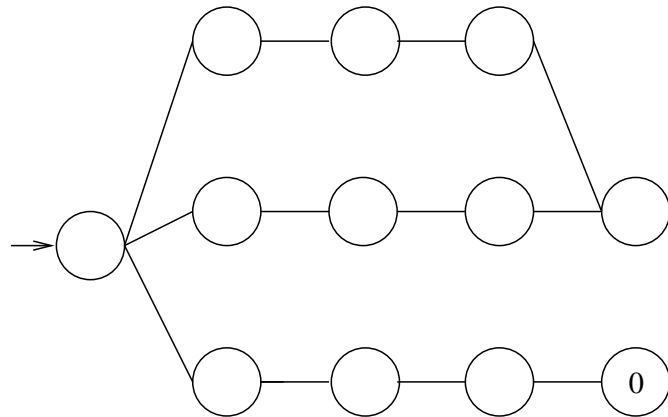
4 classes of behaviour with respect to dead ends:

Definition

A STRIPS task and its state space are called

1. **undirected**, if  $\forall (s, s') \in E : (s', s) \in E$ ,
2. **harmless**, if  $\exists (s, s') \in E : (s', s) \notin E$ , and  $\forall s \in S : sd(s) < \infty$ ,
3. **recognized**, if  $\exists s \in S : sd(s) = \infty$ , and  $\forall s \in S : sd(s) = \infty \Rightarrow h(s) = \infty$ ,
4. **unrecognized**, if  $\exists s \in S : sd(s) = \infty \wedge h(s) < \infty$ .

Undirected/harmless/recognized/unrecognized:



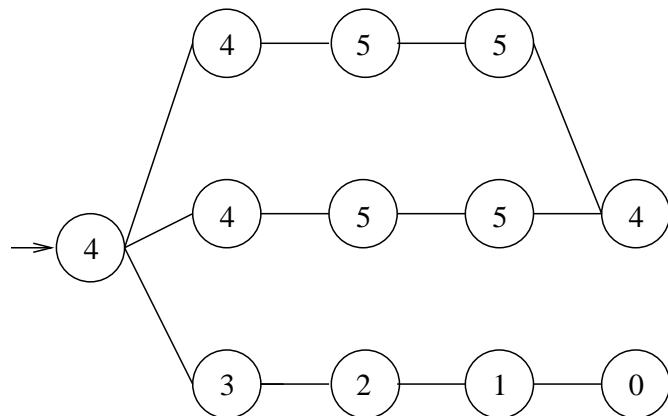
A plateau is a region of states in  $S$  that all look the same:

**Definition**

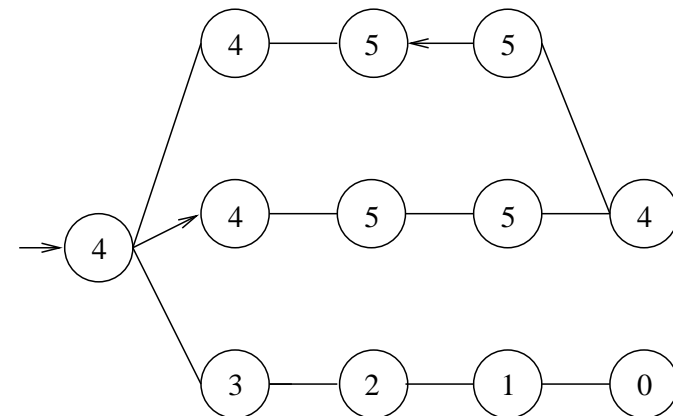
For  $l \in \mathbf{N} \cup \{\infty\}$ , a *plateau of level  $l$*  is a maximal subset  $P$  of  $S$  for which the induced subgraph in  $(S, E)$  is strongly connected, and  $h(s) = l$  for each  $s \in P$ .

- ▶ Plateaus look the same both in terms of reachability, and in terms of heuristic value
- ▶ They form the smallest entity in the subsequent definitions and investigations

What are the plateaus here?



What are the plateaus here?



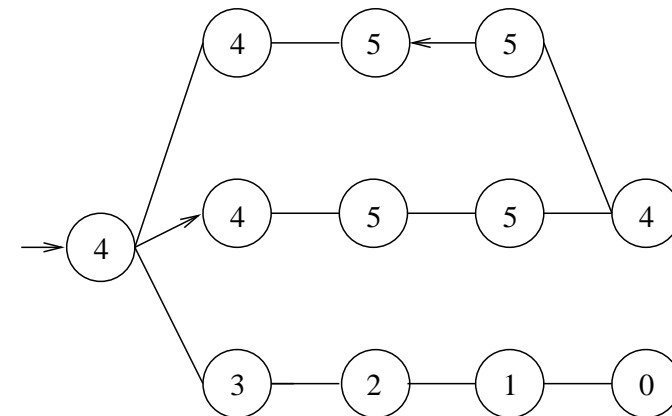
We distinguish between plateaus with the following defs:

### Definition

A path of connected states in  $S$  is *flat* if all states on it have the same  $h$  value. For a plateau  $P$  of level  $l$ , an *exit* (of  $P$ ) is a state  $s$  reachable from  $P$  on a (potentially empty) flat path, such that  $h(s) = l$  and there exists a state  $s'$ ,  $(s, s') \in E$ , with  $h(s') < h(s)$ .

- ▶ The standard definition [Frank et al,JAIR-1997] says that the exit lies *on* the plateau itself
- ▶ This is for undirected graphs; there, with the above definition any exit  $s$  of  $P$  indeed has  $s \in P$  – *Exercise*
- ▶ In directed graphs, what matters is if we can escape the plateau without temporarily worsening the heuristic value

Give me a plateau and its exit:

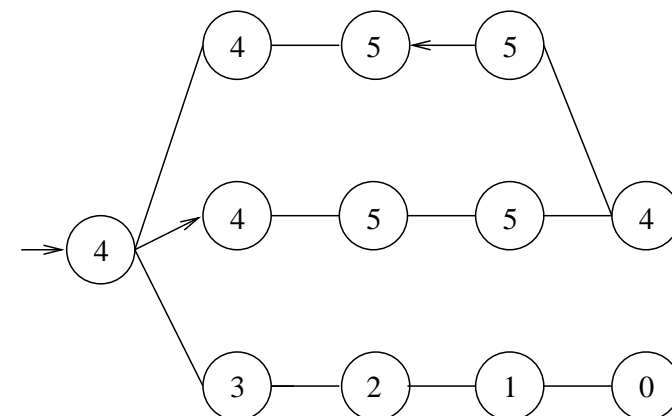


### Definition

We distinguish 5 classes of plateaus:

1. A *recognized dead end* is a plateau  $P$  of level  $\infty$ .
2. A *local minimum* is a plateau  $P$  of level  $0 < l < \infty$  that has no exits.
3. A *bench* is a plateau  $P$  of level  $0 < l < \infty$  that has at least one exit, and where at least one state on  $P$  is not an exit.
4. A *contour* is a plateau  $P$  of level  $0 < l < \infty$  that consists entirely of exits.
5. A *global minimum* is a plateau  $P$  of level 0.

What are the classes of all the plateaus?



Under the Manhattan Distance heuristic  $h^{MD}$ , what kind of plateau do we have here?

1	2	3
6	7	
4	8	9

Under the Manhattan Distance heuristic  $h^{MD}$ , what kind of plateau do we have here?

1	2	3
6	7	9
4	8	

There aren't any benches in the 8-puzzle under  $h^{MD}$  – why?

For a heuristic or local search, the main difficulty is to *escape local minima and benches*

The difficulty of doing this can be assessed by, e.g.: ???

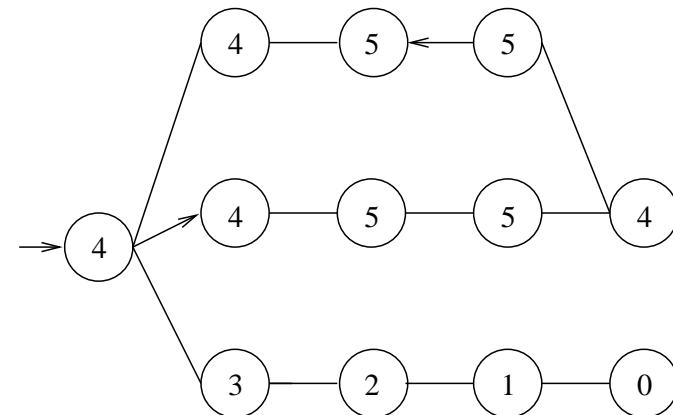
For benches the following will be relevant:

**Definition**

For a state  $s \in S$  on a bench, the **exit distance** is the length of the shortest path to an exit. The **maximal exit distance** is the maximum over all  $s$  on benches, or 0 if there aren't any benches.

“How far do we need to go (in the worst case) to find an exit?”

Give me the exit distances of the states on benches:



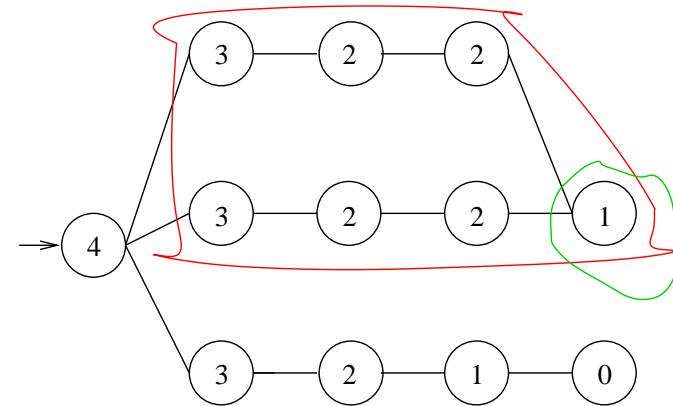
There are various easy-to-see implications. An important one that we will use:

### Proposition

If there exists a dead state  $s \in S$  with  $h(s) < \infty$ , then there exists a local minimum  $P \subseteq S$ .

### Proof.

Blackboard. □



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## 20 Benchmark Domains

I will show local search topology results for a large set of benchmark domains including all IPC-1 and IPC-2 examples.

A rough classification:

- ▶ **Transportation domains**: locations, vehicles, transportable objects. E.g. Logistics
- ▶ **Construction domains**: build a complex object out of its individual parts. E.g. Blocksworld, also Towers of Hanoi
- ▶ **Others**: ...

(I also got theoretical results for the 10 IPC-3 and IPC-4 domains, but not the empirical results that we will consider.)

- ▶ **Logistics, Gripper, Ferry**: simple transportation, with different vehicles, sometimes with capacity restrictions
- ▶ **Briefcaseworld**: objects inside the briefcase are moved around via a conditional effect
- ▶ **Driverlog**: Logistics on a road map graph and with drivers
- ▶ **Grid**: key transportation on a grid map with locked positions of different shapes
- ▶ **Miconic-STRIPS, Miconic-SIMPLE, Miconic-ADL**: elevator control domains, passengers indicate where they are and wanna go; in “-ADL” with side constraints, VIPs must be served first etc.
- ▶ **Zenotravel**: Logistics with fuel usage, refueling operator
- ▶ **Mystery, Mprime**: Logistics on road map, fuel usage, no refueling. Mprime: transfer fuel between locations



- ▶ **Simple-Tsp**: TSP on a fully connected graph with uniform edge costs. Don't laugh – Graphplan actually dies on this. *Fully automatic* can be tough even if the domain happens to be simple. (Some guys have constructed the domain to show off the performance of their symmetry detection algorithm.)
- ▶ **Movie**: buy some snacks and rewind the tape
- ▶ **Tireworld**: replace  $n$  flat tires (jack up, remove wheel, ...)
- ▶ **Fridge**: replace  $n$  broken compressors (unfasten screw, remove backplane, ...)
- ▶ **Schedule**: process (paint etc.) objects on machines



- ▶ **Blocksworld-arm, Blocksworld-no-arm**: sequential version, and parallel version moving blocks directly
- ▶ **Depots**: combines Blocksworld-arm with Logistics
- ▶ **Freecell**: the card game
- ▶ **Hanoi**: Towers of
- ▶ **Assembly**: a complex object must be assembled out of its parts, obeying ordering constraints etc.



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Build the entire state spaces of example tasks, annotate them with the heuristic values, and measure topology parameters.

- ▶ Only possible for small instances
- ▶ Parameters: e.g. percentage of unrecognized dead ends, average size of valleys, ...

In what follows:

- ▶ 100 random instances of every domain (except special cases like Gripper)
- ▶ Summarize (average) data as function of “task size” as given by nr. of objects (trucks etc.)

Percentage of states with  $h^+ < \infty$ :

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Assembly</i>	100.0	99.8	99.5	99.0	99.9
<i>Freecell</i>	100.0			98.6	97.7
<i>Miconic-ADL</i>	100.0		88.5	90.9	84.7
<i>Mprime</i>	68.1	56.1	36.6	30.7	43.3
<i>Mystery</i>	52.7	36.3	28.8	21.4	14.1
<i>Schedule</i>	55.1	63.7	67.7	97.7	97.0

We call this the “relevant” state space. All others: 100%.

No differences here to  $h^{FF}$  since  $h^{FF} = \infty \Leftrightarrow h^+ = \infty$ .

Percentage, in relevant state space, of states with  $sd = \infty, h^+ < \infty$  (unrecognized dead ends):

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Freecell</i>	0.0			0.4	2.8
<i>Miconic-ADL</i>	0.0		0.0	5.9	6.9
<i>Mprime</i>	13.3	37.9	55.5	41.5	58.8
<i>Mystery</i>	15.7	32.1	48.9	58.6	36.6

All other 16 domains: none.

Percentage, in relevant state space, of states located on valleys under  $h^+$ :

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Blocksworld-arm</i>	0.0	21.0		31.1	47.5
<i>Freecell</i>	0.0			0.4	2.8
<i>Miconic-ADL</i>	0.0		0.0	5.9	6.9
<i>Mprime</i>	13.3	38.3	56.2	42.2	65.9
<i>Mystery</i>	15.7	32.6	49.6	60.9	37.0
<i>Schedule</i>	24.8	32.6	33.1	24.3	19.9

What is the most significant observation here?

Percentage, in relevant state space, of states located on valleys under  $h^{FF}$ :

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Assembly</i>	0.0	0.0	0.0	0.0	0.2
<i>Blocksworld-arm</i>	0.0	21.0		31.1	47.5
<i>Briefcaseworld</i>	0.0		0.3		0.9
<i>Freecell</i>	0.0			0.9	2.8
<i>Grid</i>	2.6	4.2			3.1
<i>Hanoi</i>	0.0	0.0	0.0	33.2	77.7
<i>Miconic-ADL</i>	0.0		0.2	6.5	8.7
<i>Miconic-SIMPLE</i>	0.0	0.3	2.9	0.6	0.4
<i>Mprime</i>	13.3	38.4	56.1	42.2	65.9
<i>Mystery</i>	15.7	32.5	49.4	60.5	37.0
<i>Schedule</i>	24.2	32.2	32.3	24.3	18.8

“New” domains: Assembly, Briefcaseworld, Grid, Hanoi, Miconic-SIMPLE.

Maximal exit distance under  $h^+$ , excerpt:

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Blocksworld-arm</i>	1.0	3.0		3.5	4.9
<i>Blocksworld-no-arm</i>	0.0	1.0		1.5	2.4
<i>Ferry</i>	1.0	1.0	1.0	1.0	1.0
<i>Fridge</i>	2.5	4.5	4.0	6.0	7.7
<i>Gripper</i>	1.0	1.0	1.0	1.0	1.0
<i>Hanoi</i>	3.0	7.0	15.0	31.0	63.0
<i>Logistics</i>	1.0	1.0	1.0		1.0
<i>Miconic-ADL</i>	1.0		1.0	1.1	1.5
<i>Miconic-SIMPLE</i>	1.0	1.0	1.0	1.0	1.0
<i>Miconic-STRIPS</i>	1.0	1.0	1.0	1.0	1.0
<i>Movie</i>	1.0	1.0	1.0	1.0	1.0
<i>Simple-Tsp</i>	0.0	0.0	0.0	0.0	0.0
<i>Tireworld</i>	6.0				

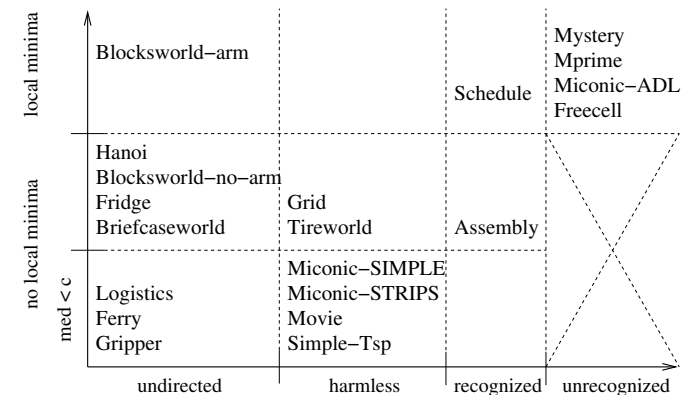
In the other domains, inconclusive or slightly growing.



Maximal exit distance under  $h^{FF}$ , excerpt:

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Blocksworld-arm</i>	1.0	3.0		3.5	4.9
<i>Blocksworld-no-arm</i>	0.0	1.0		1.3	2.4
<i>Ferry</i>	1.0	1.0	1.0	1.0	1.0
<i>Fridge</i>	2.5	4.5	4.0	6.0	7.7
<i>Gripper</i>	1.0	1.0	1.0	1.0	1.0
<i>Hanoi</i>	3.0	9.0	23.0	23.0	23.0
<i>Logistics</i>	1.0	1.0	1.0		1.0
<i>Miconic-ADL</i>	1.0		1.0	1.1	1.1
<i>Miconic-SIMPLE</i>	1.0	1.0	1.0	1.0	1.0
<i>Miconic-STRIPS</i>	1.0	1.0	1.0	1.0	1.0
<i>Movie</i>	1.0	1.0	1.0	1.0	1.0
<i>Simple-Tsp</i>	0.0	0.0	0.0	0.0	0.0
<i>Tireworld</i>	6.0				

$h^+$  behaves this way:



...and  $h^{FF}$  behaves similarly.



Proposition

Given a solvable STRIPS task  $(P, A, I, G)$  with heuristic function  $h$  so that there are no local minima, and the maximal exit distance is  $d$ , enforced hill-climbing finds a solution after evaluating  $O(|A|^{d+1} * h(I))$  states.

Proof.

Blackboard. □

Given a domain with no local minima, and constant  $c \geq d$ , enforced hill-climbing performs in the order of  $(c + 1) * h(I)$  – polynomial if  $h(I)$  isn't exponential.

Does this prove that enforced hill-climbing is polynomial in the domains in the lowermost classes of the taxonomy?

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Proving Local Search Topology

Prove, from the structural properties of a sub-class of instances, topological properties of a heuristic function.

- ▶ “Sub-class”: in particular a planning domain
- ▶ In what follows, we verify the hypothesis for  $h^+$
- ▶ Proofs domain structure  $\Rightarrow$  no (unrecognized) dead ends, no local minima, maximal exit distance bound
- ▶ We only show some core lemmas, and a few examples
- ▶ Ideally, we would want criteria testable (quickly) by a machine – so we stick as closely to “syntax” as possible



Does  $S$  contain Dead Ends?

Definition

Let **DEAD-END** denote the following decision problem. Given a STRIPS task  $(P, A, I, G)$ , does  $S$  contain a dead end?

Theorem

DEAD-END is **PSPACE**-complete.

Proof.

Membership: guess a state  $s$  and verify in PSPACE (PLANSAT for  $(P, A, I, s)$ , PLANSAT for  $(P, A, s, G)$ ) if it is in  $S$  and a deadend. Finished with  $\text{NPSPACE} = \text{PSPACE} = \text{co-PSPACE}$ .  
Hardness: Exercise. □



We will derive two sufficient criteria. We need:

### Definition

Let  $(P, A, I, G)$  be a STRIPS task. Two facts are *inconsistent* if there is no reachable state  $s \in S$  that contains both of them. A set of facts  $F$  *excludes* a set of facts  $F'$  if each fact in  $F'$  is inconsistent with at least one fact in  $F$ .

“excludes” is *not* a symmetric definition:  $\{at(A, L), at(T, L)\}$  excludes  $\{in(A, T)\}$  but not vice versa!

Means: if  $F$  is true, then none of  $F'$  is true.

(There exist many good – typically fast and informative – approximation techniques for inconsistency: e.g., 2-planning graphs.)

### Definition

Let  $(P, A, I, G)$  be a STRIPS task. An action  $a$  is *invertible* if:

1. there is an action  $\bar{a} \in A$  such that
  - 1.1  $pre(\bar{a}) \subseteq (pre(a) \cup add(a)) \setminus del(a)$
  - 1.2  $add(\bar{a}) = del(a)$
  - 1.3  $del(\bar{a}) = add(a)$
2.  $pre(a)$  excludes  $add(a)$
3.  $del(a) \subseteq pre(a)$ .

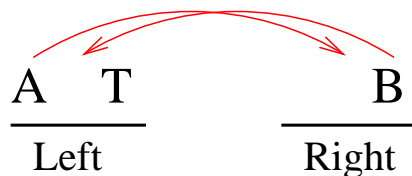
### Lemma

Let  $(P, A, I, G)$  be a STRIPS task. If all actions  $a \in A$  are invertible, then the state space to the task is undirected.

### Proof.

Blackboard. □

## Example Logistics



Blackboard.

## At Least Invertible Actions

### Definition

Let  $(P, A, I, G)$  be a STRIPS task. An action  $a$  is *at least invertible*, if there is an action  $\bar{a} \in A$  such that

1.  $pre(\bar{a}) \subseteq (pre(a) \cup add(a)) \setminus del(a)$ ,
2.  $add(\bar{a}) \supseteq del(a)$ , and
3.  $pre(a)$  excludes  $del(\bar{a})$ .

“At least”:  $res(s, \langle a, \bar{a} \rangle) \supseteq s \dots!$

Example “Jog”:  $jog(x, y) = (\{at(x)\}, \{at(y), did-jog()\}, \{at(x)\})$

If  $a$  is invertible then  $a$  is also at least invertible: Blackboard.

**Definition**

Let  $(P, A, I, G)$  be a STRIPS task. An action  $a$  has irrelevant delete effects, if

$$\text{del}(a) \cap (G \cup \bigcup_{a' \neq a \in A} \text{pre}(a')) = \emptyset$$

An action has static add effects, if

$$\text{add}(a) \cap \bigcup_{a' \in A} \text{del}(a') = \emptyset$$

Example Tireworld:  $\text{inflate}(\text{wheel})$  :  
 $(\{\text{NOTinflated}(\text{wheel})\}, \{\text{inflated}(\text{wheel})\}, \{\text{NOTinflated}(\text{wheel})\})$

**Lemma**

Let  $(P, A, I, G)$  be a solvable STRIPS task. If for all  $a \in A$  either

1.  $a$  is at least invertible, or
2.  $a$  has irrelevant delete effects and static add effects,

then  $S$  contains no dead ends.

**Proof.**

Action sequence  $a_1, \dots, a_n$  leading to state  $s$ , plan for initial state  $p_1, \dots, p_m$ . Construct plan for  $s$  as follows:

```

M := ∅
for i := n ... 1 do
  if a_i is at least invertible by ā_i then
    if ā_i ∉ M apply ā_i endif
  else M := M ∪ {a_i}
  endif
endifor
for i := 1 ... m do
  if p_i ∉ M then apply p_i endif
endifor

```

## Example Tireworld

- ▶ All actions ( $\text{fasten}(\text{screw}), \text{unfasten}(\text{screw}), \dots$ ) are invertible, except  $\text{inflate}(\text{wheel})$
- ▶ Say  $s$  is reached via  $a_1, \dots, a_n$ , and  $p_1, \dots, p_m$  is a plan for the initial state
- ▶ “Undo  $a_1, \dots, a_n$  except for the  $\text{inflate}(\text{wheel})$  actions; store the latter in  $M$ ”
- ▶ “We are back to the initial state, except that some wheels – corresponding to  $M$  – are already inflated”
- ▶ “Execute  $p_1, \dots, p_m$  except for inflating the wheels –  $M$  – that already are inflated”

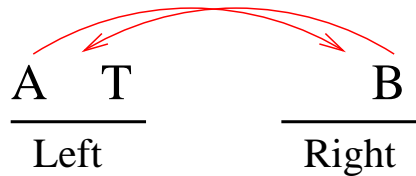
Does S contain Local Minima under  $h^+$ ?**Definition**

Let  $(P, A, I, G)$  be a STRIPS task. An action  $a$  is **respected by the relaxation** if, for any reachable state  $s \in S$  such that  $a$  starts an optimal plan for  $(P, A, s, G)$ , there is an optimal relaxed plan for  $(P, A, s, G)$  that also starts with  $a$ .

**Definition**

Let  $(P, A, I, G)$  be a STRIPS task. An action  $a$  is **at least relaxed invertible**, if there is an action  $\bar{a} \in A$  such that  $\text{pre}(\bar{a}) \subseteq (\text{pre}(a) \cup \text{add}(a)) \setminus \text{del}(a)$ , and  $\text{add}(\bar{a}) \supseteq \text{del}(a)$ .

This is yet weaker than “invertible” and “at least invertible”.



Blackboard.

Lemma

Let  $(P, A, I, G)$  be a solvable STRIPS task whose state space does not contain unrecognized dead ends. If each action  $a$

1. is respected by the relaxation, and
2. has irrelevant delete effects or is at least relaxed invertible,

then there are no local minima under evaluation with  $h^+$ .

Proof.

Blackboard.

Main line to remember: We can obtain a relaxed plan for  $res(s, a)$  by either removing  $a$  from  $P^+$  for  $s$ , or replacing it with  $\bar{a} \dots!$  □

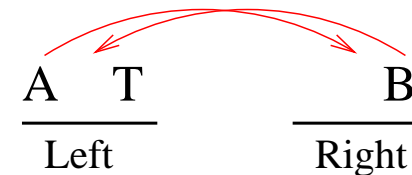
What is the Maximal Exit Distance under  $h^+$ ?

Example Logistics

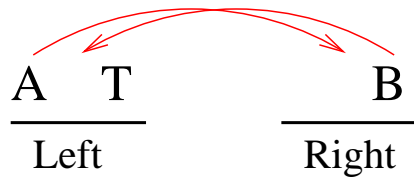
There is (apparently) no nice “syntactic” criterion. Best I could find is this:

An action  $a$  has **solution-irrelevant delete effects** if, when  $a$  starts an optimal plan for  $s$  yielding state  $s'$ , there is an optimal plan for  $s'$  that does not need the facts deleted by  $a$ .

Say the prerequisites of the previous lemma hold. If, for any state  $s$ , there is an optimal plan for  $s$  that starts with at most  $d$  actions with non solution-irrelevant delete effects, then  $d$  is an upper bound on the maximal exit distance. *Do you see why?*

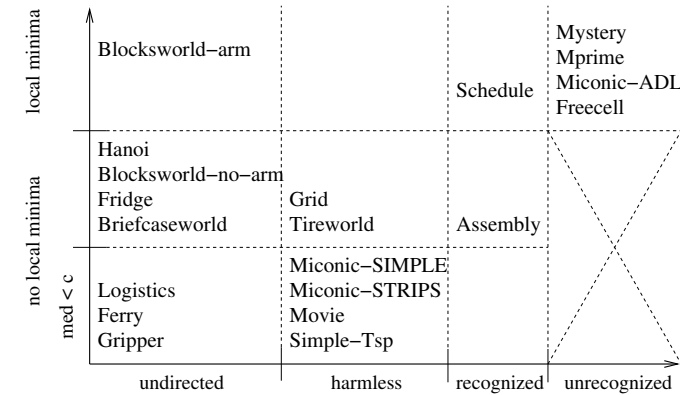


Blackboard.



1. The state space is ???
2. There ??? local minima
3. The maximal exit distance is bounded by ???

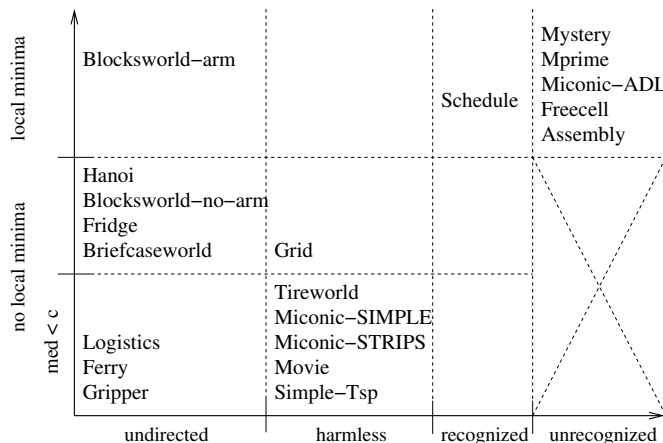
$h^+$  behaves this way:



The Proved Taxonomy

“Example” Assembly

$h^+$  behaves this way:



This is a little more complicated.

Lemma

Given an instance of the Assembly domain, and a state  $s$ , such that for all relevant leafs  $x$  in  $NAG(s)$ ,  $(available, x) \in s$ . If

1.  $(As, NA(s) \cup AC(s))$  is cycle-free, and
2. for all  $(x, y) \in NA(s)$ , there is a sequence  $x = x_0, x_1, \dots, x_n$  such that  $x_n \in pa(y)$ ,  $(x_{i-1}, x_i) \in ao(y)$  and  $(x_i, y) \in NA(s)$  for  $1 \leq i \leq n$ ,

then  $s$  does not lie on a valley.

The good news: we don't consider this here.

- ▶ A Reminder:  $h^+$  and  $h^{FF}$
- ▶ A Preview
- ▶ Local Minima & Co.
- ▶ 20 Planning Benchmark Domains
- ▶ Measuring Local Search Topology
- ▶ Proving Local Search Topology
- ▶ **Sampling Local Search Topology**

Generate random states in large examples, measure topological properties of these states, do (simple) statistics.

- ▶ This is the only thing anybody previously did (in SAT [Frank et al, JAIR 1997])
- ▶ Only way to get empirical data about topology of large state spaces; applicable in any context

Here: verify *to what extent  $h^+$  topology transfers to  $h^{FF}$*

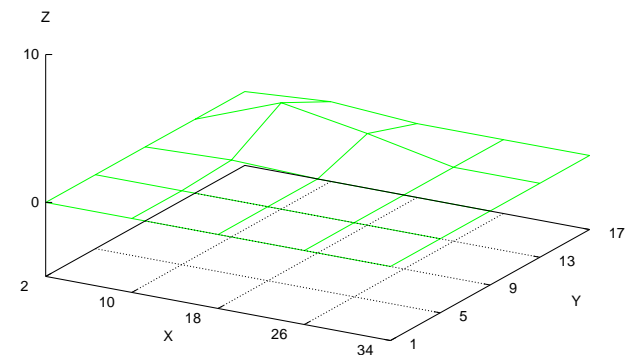
- ▶ For “no local minima” and “med  $\leq c$ !” (dead end behavior provably identical)
- ▶ We plot topological data against **domain parameters**

Average percentage of sample states on valleys:

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Blocksworld-no-arm</i>	0.0	0.0	0.0	0.1	0.0
<i>Gripper</i>	0.0	0.0	0.0	0.0	0.0
<i>Hanoi</i>	0.0	0.0	96.0	100.0	100.0
<i>Movie</i>	0.0	0.0	0.0	0.0	0.0
<i>Simple-Tsp</i>	0.0	0.0	0.0	0.0	0.0
<i>Tireworld</i>	0.0	0.0	0.0	0.0	0.0

(domains with 1 domain parameter)

Average percentage of sample states on valleys:



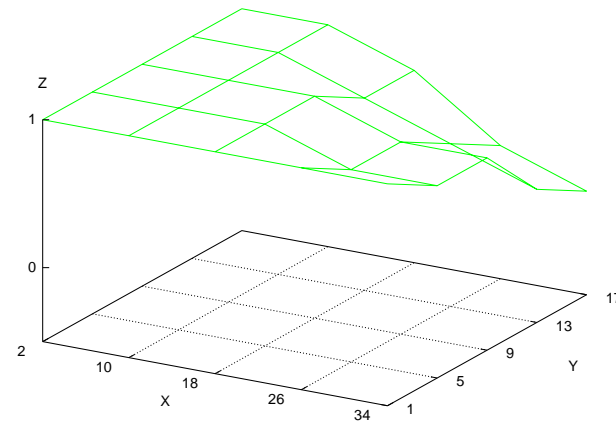
Scaling floors  $x$  against passengers  $y$  in Miconic-SIMPLE.

Average maximal exit distance of sample states:

Domain	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$
<i>Gripper</i>	1.0	1.0	1.0	1.0	1.0
<i>Movie</i>	1.0	0.9	0.6	0.4	0.6
<i>Simple-Tsp</i>	0.0	0.0	0.0	0.0	0.0
<i>Tireworld</i>	6.0	6.0	6.0	6.0	2.0
<i>Blocksworld-no-arm</i>	0.3	1.8	2.8	3.8	3.7
<i>Hanoi</i>	6.0	23.0	12.0	2.0	2.0

(domains with 1 domain parameter)

Average maximal exit distance of sample states:



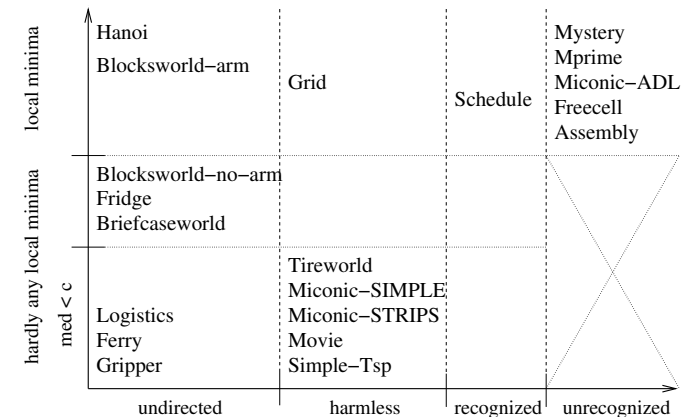
Scaling floors  $x$  against passengers  $y$  in Miconic-SIMPLE.



... this way, becomes pretty ... when a domain has, say, more than 5 domain parameters.

Anyway:

$h^{FF}$  behaves this way:



$h^+$  is an extremely informative heuristic in many Planning benchmarks.

In particular we can conclude that FF is “largely polynomial” in Gripper, Ferry, Logistics, Simple-Tsp, Movie, Miconic-STRIPS, Miconic-SIMPLE, and Tireworld.

(As well as Zenotravel, Satellite, Schedule, and Dining-Philosophers.)

Most important question: **Does this phenomenon occur also in real domains?** (Put in another way: “Is this good or bad news?”)

My intuition: “yes, but only in some domains, and not in such an extreme form”

- ▶ Jeremy Frank, Peter Cheeseman, and John Stutz, *When Gravity Fails: Local Search Topology*, JAIR, 1997.
- ▶ Jörg Hoffmann, *Local Search Topology in Planning Benchmarks: An Empirical Analysis*, IJCAI 2001.
- ▶ Jörg Hoffmann, *Local Search Topology in Planning Benchmarks: A Theoretical Analysis*, AIPS 2002.
- ▶ Jörg Hoffmann, *Utilizing Problem Structure in Planning: A Local Search Approach*, LNAI 2854, Springer-Verlag, 2003.
- ▶ Jörg Hoffmann, *Where “Ignoring Delete Lists Works: Local Search Topology in Planning Benchmarks”*, JAIR, 2005.